Integrated Inertial Reference – Inertial Navigation System

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The value of Inertial Navigation Systems (INS) is reduced for long time-duration flights because uncertainties produce increased position errors with time. A reduction in this position error buildup is obtained from a mechanization concept described in this paper. This mechanization comprises an Electrically Suspended Gyroscope (ESG) mounted on the stabilized platform of a local-level INS. Position is estimated by tracking the local vertical while accounting for the Earth's rotation with an accurate clock. Subsystems are integrated by means of a Kalman-Bucy filter. An accuracy of 0.2 n. mi./hr for 12 hr of flight was obtained in a simulation study using an ESG and a 1 n. mi./hr INS.

Introduction

INERTIAL Navigation Systems (INS) are especially useful for long-range flights and military applications because they are self-contained devices, operable in nearly any location and not susceptible to interference. Their value is reduced for long time-duration flights, however, because system uncertainties produce an increased position error with time.

The effect of the self-contained inertial reference-INS mechanization described is a reduction in this error buildup. The basic system implementation discussed comprises an Electrically Suspended Gyroscope (ESG) inertial reference mounted on the stabilized platform of a local-level INS, accurate clock, and computer. The inertial reference can be of any type inertial quality gyroscope, it need not be an ESG. The only requirement is that it provide high quality space stabilization.

At alignment, the inertial reference and local level INS stable platform upon which it is mounted are brought into coincidence with the initial local vertical. During navigation, the inertial reference maintains the orientation of the initial local vertical in inertial space. The clock accounts for the angular motion of the initial local vertical due to the Earth's rotation. The local-level INS stable platform, which supports the inertial reference, is continually rotated into coincidence with the present local vertical. The present local vertical is unique to the present geographic Earth location.

The inertial reference-INS therefore provides a measure of the change in the local vertical in inertial space due to transport motion (motion of the system with respect to the Earth) and the rotation of the Earth with respect to inertial space. Position can be estimated from this angular change in the local vertical in inertial space and from knowledge of the initial position. This estimate of position is used as information to update the local-level INS.

In actual application, the inertial reference, local-level INS and clock are integrated by means of a Kalman-Bucy filter. Components of change in the local vertical in inertial space due to transport motion and the Earth's rotation are computed from INS measurements of position as well as a knowledge of the initial position. These computed values are compared with values measured by the inertial reference-INS and clock. The resulting error quantities are operated on by the filter equations which yield optimal estimates of position and other quantities in a minimum mean square error sense. Filter equations, as well as conventional INS navigation and platform management computations, are solved in the computer.

This paper discusses the principles involved in the inertial reference-INS mechanization. Local vertical measurements are considered, and position determination by tracking the local vertical is explained. System hardware implementation and theory of operation are also detailed. A Kalman-Bucy filter is described which enables optimum estimates of the subsystem error quantities to be made. Finally, a detailed account is given of a simulation study conducted to evaluate inertial reference-INS performance over a long time-duration flight profile. Results of the study are presented.

Inertial Navigation Systems

Dynamically, the INS is an undamped, second-order, open-loop system. System uncertainties cause output errors which are oscillatory and increase with time. Figure 1 shows a typical time history of the error in position indication of an INS resulting from system uncertainties. INS position errors can be bounded by updating with position information from an external source. Figure 1 shows a position error vs time curve for an updated INS.

The objective of the mechanization described in the following section is to reduce the buildup in position error by periodic updating from an internal source. The internal source comprises a low-drift ESG and an accurate clock.

An Inertial Reference – Inertial Navigation System

General

Every local vertical is unique to the geographic Earth location. The local vertical cannot be identified in inertial space, unless an inertial reference is established in time and space. The inertial reference – inertial navigation system discussed here is mechanized to identify the local vertical in inertial space and thereby reduce the buildup of the position error. The system takes advantage of the inherent accuracy of the local-level INS in maintaining the local vertical. The implementation also includes an accurate clock and low-drift ESG for establishing an inertial reference in time and space.

The Local Vertical

The inertial reference-INS is designed to report position by tracking the angular displacement of the gravity vector. The gravity vector is the vector sum of the Earth's gravitational and centrifugal fields. The local vertical corresponds to the direction of the gravity vector and is therefore subject to gravity anomalies. Passing over an irregularity in the Earth's gravitational field will have a deleterious but transitory effect on the inertial reference – INS.

The local vertical is associated with the longitude and astronomic latitude of a point on the surface of the Earth. The

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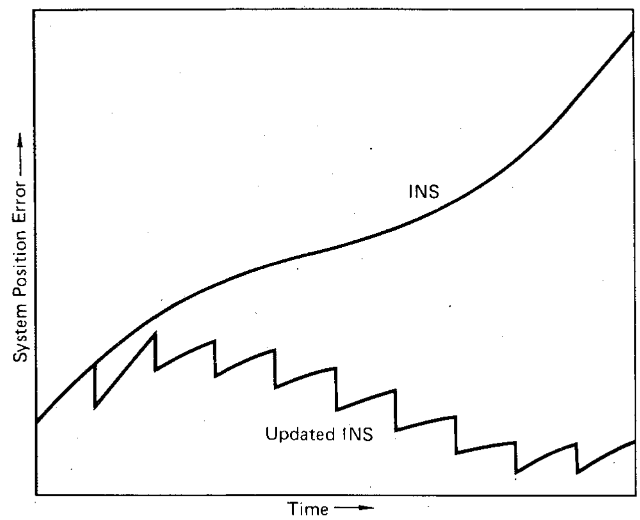

Fig. 1 Typical system position error propagation.

usual understanding of map latitude is geodetic latitude. The normal to the best-fitting body of rotation around the Earth's spin axis chosen to represent the sea-level equipotential surface of the Earth is associated with the longitude and geodetic latitude of a point on that surface. The best-fitting ellipsoid is chosen to minimize the mean square deviation between the local vertical and the normal to the ellipsoid. The deflection of the vertical, the angular deviation between the local vertical, and the normal to the best-fitting ellipsoid, is usually less than 10 sec of arc and infrequently greater than 30 sec of arc.[1] Consequently, local vertical latitude and map latitude are approximately the same. Inertial navigation systems retain an accurate track of the local vertical.

An accurate clock and low-drift inertial reference are required in mechanizing the inertial reference-INS. Oven-controlled crystal oscillator clocks are currently available with an accuracy of one part in 10^7 and drift rates of five parts in 10^{10} per day (long term).

The low-drift ESG is capable of establishing an inertial reference in space. It is a two degree-of-freedom gyroscope in which the rotor is supported by an electric field. The absence of an axle and bearings eliminates sources of large random drift rates. Part of the small drift rates that remain is predictable.

Position Determination by Tracking Angular Displacement of the Local Vertical

Position is determined by tracking the local vertical using transformation relationships between spherical and rectangular coordinate systems. If $\bar{i}$, $\bar{j}$ and $\bar{k}$ are unit vectors in the rectangular coordinate system described in Fig. 2, and La_n and Lo_n denote the latitude and longitude of a point n on a spherical surface, then $\overline{1_n}$ is the unit vector normal to the surface at n and given by

$$\overline{1_n} = \cos La_n \cos Lo_n \bar{i} + \cos La_n \sin Lo_n \bar{j} + \sin La_n \bar{k} \qquad (1)$$

The point I In Fig. 2 represents the initial point on a spherical surface with latitude and longitude coordinates La_I and Lo_I, respectively. The unit vector normal to I is designated $\overline{1_I}$. The point p corresponds to the present position on the same spherical surface with coordinates La_p and Lo_p. The unit vector normal to the present position is $\overline{1_p}$. A is the angular distance between I and p.

The point Ip in Fig. 2 is determined by traversing the surface over an angular distance B of constant latitude La_I from longitude Lo_I to longitude Lo_p. The same point is also reached by moving over an angular distance C of constant longitude Lo_p from latitude La_p to latitude La_I. The unit vector $\bar{i}_{Ip}$ is normal to the spherical surface at Ip.

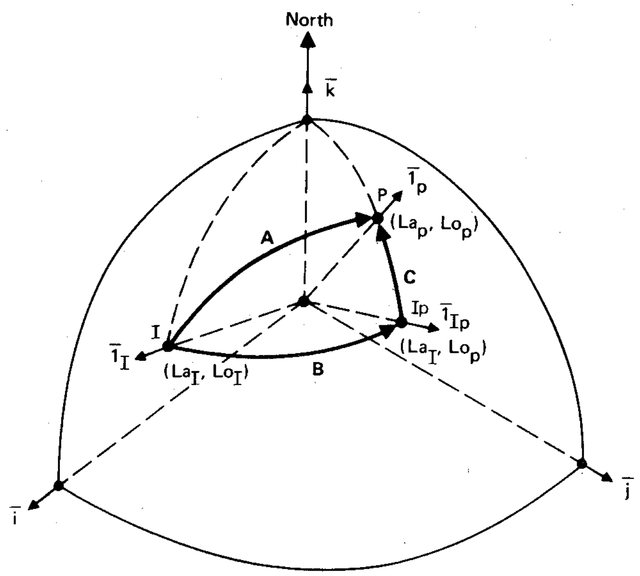

Fig. 2 Position determination by tracking the local vertical.

The angular distances B and C are obtained by solving for the inverse cosine of the inner products of $\overline{1_I}$ and $\overline{1_{Ip}}$, and $\overline{1_p}$ and $\bar{i}_{Ip}$, respectively

$$B = \cos^{-1}(\cos^2 La_I \cos(Lo_p - Lo_I) + \sin^2 La_I) \qquad (2)$$

$$C = La_p - La_I \qquad (3)$$

Equation (2) and (3) can be manipulated to show how present position is obtained from known initial position coordinates, La_I and Lo_I, and measured angular distances B and C.

$$Lo_p = Lo_I + \cos^{-1}\left[\frac{\cos B - \sin^2 La_I}{\cos^2 La_I}\right] \qquad (4)$$

$$La_p = La_I + C \qquad (5)$$

Actual position determination by tracking the local vertical on a rotating Earth involves an angular distance B' rather than B.

$$B' = \cos^{-1}\,[\cos^2 La_I \cos(Lo_p - \Omega_E \cos La_I \cdot (t_p - t_I) - Lo_I) + \sin^2 La_I] \qquad (2a)$$

where Ω_E is the magnitude of the angular velocity of the Earth, t_p is the present time and t_I is the initial time. Equation (2a) takes into account the effects of the Earth's rotation on the orientation of the initial local vertical in inertial space. Equations (2a) and (3) are used directly in the system application and described in further detail in the next section.

System Description

Implementation

A block diagram of the inertial reference-INS is given in Fig. 3. The mechanization includes an ESG mounted directly on the gimbaled platform of a conventional local-level INS with the intended axis of rotation of its rotor parallel to the platform's vertical axis and its case-oriented readout devices aligned with the north and east axes of the platform.

The system also includes a computer and an oven-controlled, compensated, crystal oscillator clock capable of being reset against a radio standard. The clock reports time elapsed. This time interval is used to account for the Earth's rotation in computing the angular displacement of the local vertical. Platform management signals and Kalman filter equations are calculated in the computer.

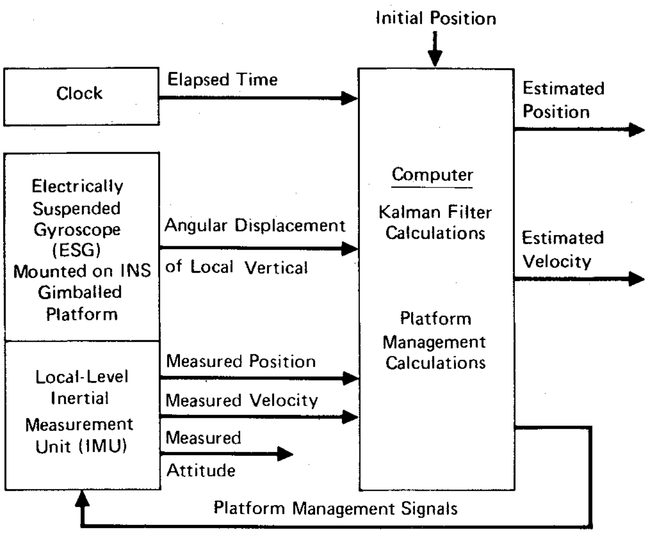

Fig. 3 Integrated inertial reference – inertial navigation system block diagram.

The Kalman filter equations process measurements of position and velocity out of the INS and angular displacements from the ESG, and yield optimal estimates of postion and other state variables in the minimum mean square error sense.

Theory of Operation

After alignment, the INS reports position, velocity, and attitude in the conventional manner, and maintains the local vertical over the flight profile. The ESG remembers the orientation of the local vertical in inertial space at the time and place of departure. The ESG pickoffs sense the angular displacements of the local vertical about the north and east axes.

The angular displacement about the north axis corresponds to B' defined in Eq. (2a). Displacement about the east axis is a measure of the angular travel in the meridianal plane and corresponds to the angular distance C formulated in Eq. (3).

Present position reported by the INS and initial position are inputs to the computer where the angular displacements B' and C are calculated using Eqs. (2a) and (3). These computed quantities are compared with the corresponding values measured by the ESG. The resulting error quantities provide a measurement of the state vector estimated by the Kalman-Bucy filter process.

The state vector comprises INS position and velocity errors, platform tilt and misalignment angles, INS instrument errors, barometric altimeter errors, ESG tilt angles and drift rates, clock uncertainty, and deflections in the vertical. The Kalman-Bucy filter process generates optimal estimates of the error state. These estimates permit updating of INS derived position and velocity quantities, and resetting of certain instrument parameters.

Kalman-Bucy Filter

A linear dynamic system is modeled in the Kalman-Bucy filter describing the response of a 23-state vector to initial conditions and inputs to the system. Components of the 23-state vector are defined in Table 1. The first nine variables, x_1 through x_p, are associated with the INS state. The equations relating the dynamic behavior of these errors are well documented[2,3] and will not be treated in this paper.

State variables x_{10} through x_{16} concern INS instruments, and x_{17} through x_{20} concern the ESG inertial reference. Clock uncertainty is represented by the state variable x_{21}. Uncertainties in the angle between the gravity vector and the normal to the reference ellipsoid are designated x_{22} and x_{23}. The time rates of change, $\dot{x}_{17}$ and $\dot{x}_{18}$, are driven by x_{19} and x_{20}, respectively. The error terms x_{10} through x_{16} and x_{19} through x_{23} are modeled as exponentially correlated noise with an rms value equal to the standard deviation and a correlation time.

Table 1 Inertial reference – inertial navigation system state vector

State Variable	Definition
x_1	Longitude Error
x_2	Latitude Error
x_3	North Velocity Error
x_4	East Velocity Error
x_5	Azimuth Error
x_6	North Axis Tilt
x_7	East Axis Tilt
x_8	Altitude Error
x_9	Vertical Velocity Error
x_{10}	Azimuth Gyro Drift Rate
x_{11}	North Gyro Drift Rate
x_{12}	East Gyro Drift Rate
x_{13}	North Accelerometer Noise
x_{14}	East Accelerometer Noise
x_{15}	Vertical Accelerometer Noise
x_{16}	Barometric Altimeter Noise
x_{17}	ESG North Axis Tilt with Respect to Platform
x_{18}	ESG East Axis Tilt with Respect to Platform
x_{19}	ESG Drift Rate About North Axis
x_{20}	ESG Drift Rate About East Axis
x_{21}	Clock Uncertainty
x_{22}	Vertical Deflection About North Axis
x_{23}	Vertical Deflection About East Axis

Kalman-Bucy Filtering Process

The set of state variables is denoted as the vector $x(t)$ at time t. Denoting the optimum (minimum mean square error) estimate of $x(t)$ immediately before obtaining a measurement $y(t)$ as $\hat{x}(t-)$, the optimum estimate of $x(t)$ after obtaining the measurement is $\hat{x}(t+)$ and is defined by Eq. (6)

$$\hat{x}(t+) = \hat{x}(t-) + k(t)\ [y(t) - H(t)\hat{x}(t-)] \tag{6}$$

where $k(t)$ is the 23×2 filter gain matrix.

The measurement vector $y(t)$ comprises two components

$$y(t) = \begin{bmatrix} C(t)_c - C(t)_M \\ B'(t)_c - B'(t)_M \end{bmatrix} \tag{7}$$

where $B'(t)$ and $C(t)$ are the angular displacements discussed previously. $B'(t)_c$ and $C(t)_c$ are values computed from INS position measurements using Eqs. (2a) and (3). $B'(t)_M$ and $C(t)_M$ are values measured by the ESG.

The linear combination of nine state variables observable through the measurement $y(t)$ in the absence of noise is given by the nonzero elements of the 2×23 observation matrix listed in Table 2. Observable state variables include longitude and latitude errors, north and east platform axes tilt angles, north and east ESG axes tilt angles with respect to the platform, clock uncertainty, and vertical deflections about the north and east axes.

The complete set of state variables, including all INS error sources, is estimated from noisy measurements of $y(t)$. The difference between the noisy measurement of $y(t)$ and the estimated measurement $\hat{y}(t) = H(t)\hat{x}(t-)$, the measurement

Table 2 Nonzero elements of the observation matrix $H(t_p)$

$$H(1,2) = 1 \quad H(2,1) = \frac{\cos^2 La_I \sin(Lo_p + \Omega_E \cos La_I (t_p - t_I) - Lo_I)}{\sqrt{1 - \cos^2 La_I \cos(Lo_p + \Omega_E \cos La_I (t_p - t_I) - Lo_I)}}$$

$H(1,7) = -1 \quad H(2,6) = -1$

$H(1,17) = -1 \quad H(2,18) = -1$

$H(1,22) = -1 \quad H(2,21) = H(2,1)\,\Omega_E \cos La_I$

$H(2,23) = -1$

La_I, Lo_I	Initial Latitude and Longitude at Time t_I
Lo_p	Present Longitude at Time t_p
Ω_E	Earth's Rate of Rotation (72.921158×10^{-6} rad/sec)

error, is pre-multiplied by the filter gain matrix $k(t)$. The resultant product is then applied as a correction factor to update the error state vector $\hat{x}(t-)$.

The elements of the $k(t)$ matrix weight the measurement error as a function of the covariance of measured and estimated state variables and the accuracy of these measurements. An unobservable state variable will be estimated accurately and therefore be significantly reduced if strongly correlated with the observable variables and if measurements are accurate. An unobservable state variable will not be significantly reduced if it is weakly correlated with the observable variables. For example, in an over 12 hr. simulated flight (discussed later and using the estimation process described above), the platform azimuth error was reduced by 70% although the azimuth gyro drift rate was reduced by only 0.7%.

Simulation Study

General

The self-contained navigation system comprising an ESG inertial reference mounted on the gyro-stabilized platform of a north-pointing, local-level INS was simulated on a digital computer. System performance was evaluated over a flight profile of a long time-duration. Results of the study indicated the feasibility of achieving 0.2 n.mi./hr from a 1 n.mi./hr INS and an ESG with a potentially attainable random drift rate of 0.0001 deg/hr (1σ).[5]

System Model

The inertial reference-INS modeled in the simulation study is described in Table 3. The set of parameters was selected to provide a reasonable model for evaluating a concept. The model was not intended to represent specific hardware items. The INS was configured to achieve 1 n.mi./hr accuracy in cruising flight. Only random errors were simulated. Systematic error sources were not included.

Convariance analyses commenced eight min before takeoff after coarse leveling and coarse alignment were assumed to be accomplished. Fine leveling went on for 30 sec followed by gyrocompassing with fine leveling for 7.5 min using a Kalman-Bucy filter to process velocity measurements. The update interval was 25 sec. Integration of the ESG inertial reference and the INS started at takeoff using a Kalman-Bucy filter update interval of 100 sec.

Flight Profile

Performance of the integrated inertial reference-INS was evaluated over a 12 hr flight profile described in Table 4. The simulated flight profile commenced with takeoff and climb to 28,000 ft altitude at a cruising speed of 355.3 knots followed by 11.5 hr of turning at 24.4 deg/min.

Turning was introduced to simulate the on-station portion of a patrol mission and to impose a heavy demand on the system. The position error grows directly as a double-time integral of the product of centripetal acceleration and misalignment in azimuth. It also grows directly as a result of tilt errors caused by the same source. A total flight duration of 12 hr,

Table 3 Inertial reference – inertial navigation system simulation study model

Exponentially Correlated Noise States			
State Variables	Definition	Standard Deviation	Correlation Time (sec)
x_{10}, x_{11}, x_{12}	INS Gyro Drift Rates	0.003 deg/hr	7,200
x_{13}, x_{14}, x_{15}	Accelerometer Noise	15 μG's	300
x_{16}	Barometric Altimeter Noise	150 ft	600
x_{19}, x_{20}	ESG Drift Rates	0.0001 deg/hr	10,800
x_{21}	Clock Uncertainty	0.5 μsec	10,800
x_{22}, x_{23}	Vertical Deflection	10 sec of Arc	1,200

Nonzero Initial Conditions		
State Variables	Definition	Values in Seconds of Arc
x_5	Azimuth Error at End of Coarse Alignment	7200
x_6	North Axis Tilt at End of Coarse Alignment	360
x_7	East Axis Tilt at End of Coarse Alignment	360
x_{17}	ESG North Axis Tilt with Respect to Platform at Takeoff	6.4
x_{18}	ESG East Axis Tilt with Respect to Platform at Takeoff	6.4

Measurement Errors	Standard Deviation
Displacements B' and C	5 sec of Arc

Table 4 Simulation study flight profile

Profile Commands				Result in Flight Profile				
Time (sec)	$\dot{V}_G$ (ft/sec^2)	$\ddot{h}$ (ft/sec^2)	$\dot{\psi}$ (deg/min)	La a) (deg/North)	Lo b) (deg/West)	h c) (ft)	ψ d) (deg)	V_G e) (kts)
0	6.0	5.0	0.0	36.07	75.66	0	315.0	0.0
20	6.0	0.0	0.0	36.07	75.66	1,000	315.0	71.0
100	0.0	0.0	0.0	36.12	75.74	8,998	315.0	355.3
280	0.0	−5.0	0.0	36.33	76.00	26,993	315.0	355.3
300	0.0	0.0	0.0	36.36	76.03	28,000	315.0	355.3
1,800	0.0	0.0	24.4	38.11	78.21	28,000	315.0	355.3
43,200	0.0	0.0	24.4	37.79	78.33	28.000	39.9	355.3

a) La Latitude
b) Lo Longitude
c) h Altitude Above Sea Level
d) ψ Heading Angle with Respect to True North
e) V_G Ground Speed

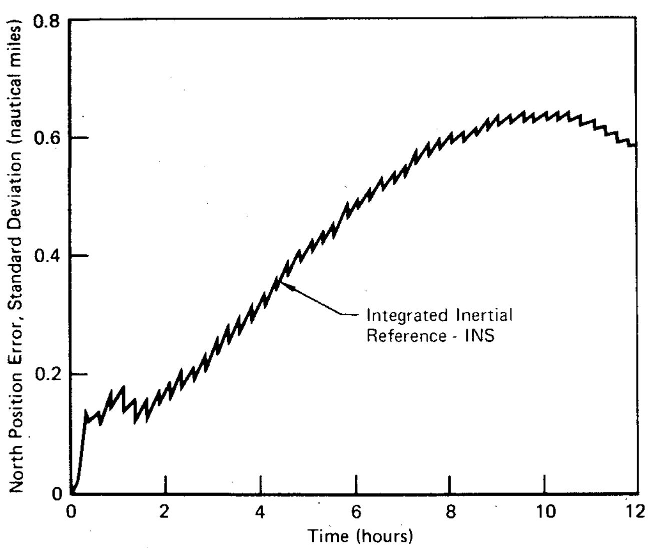

Fig. 4 North position error.

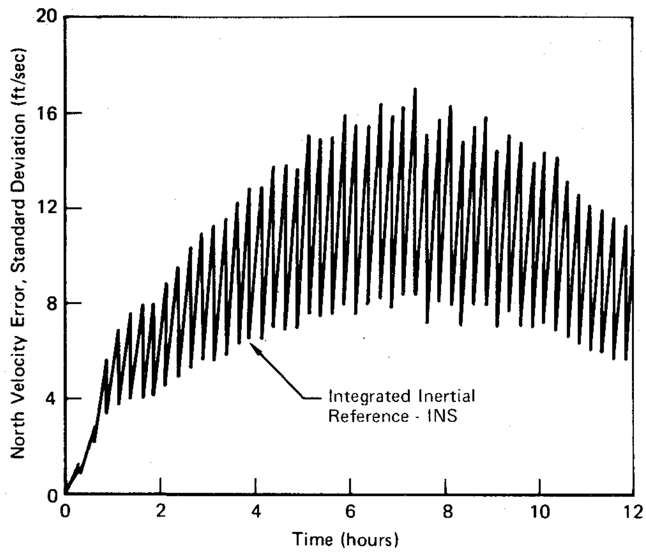

Fig. 6 North velocity error.

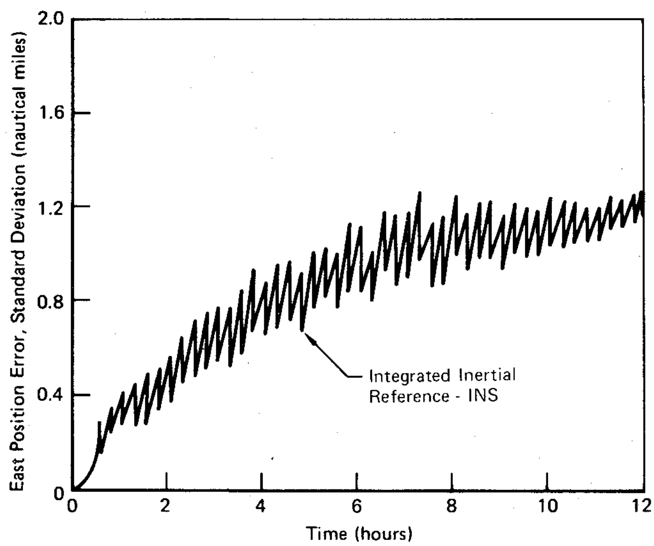

Fig. 5 East postion error.

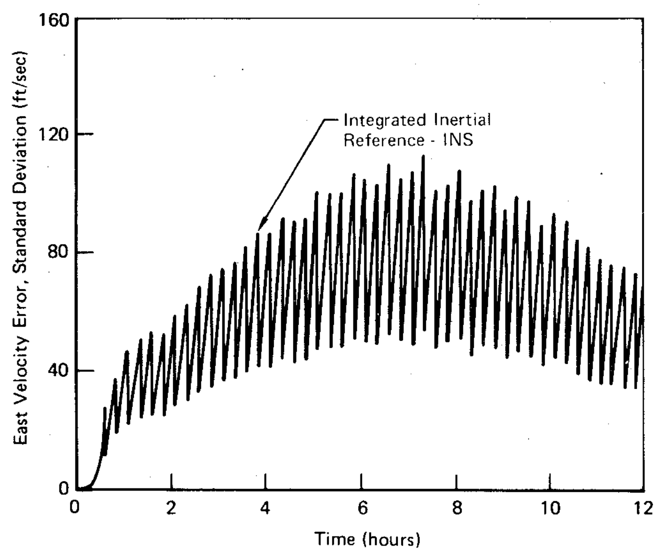

Fig. 7 East velocity error.

considerably longer than typical tactical missions, was selected to force the system to its limits.

Results

The system behaved very well throughout the 12 hr of simulated flight. The CEP (excepted radial error for an ensemble of systems) in n. mi. of the simulated inertial reference-INS was 0.3, 0.4, 0.5, 0.6 and 2.4 at the end of 2, 2.5, 3.5, 4 and 12 hr, respectively.

Figures 4 and 5 give time histories of the north and east position errors. North and east velocity errors are presented in Figs. 6 and 7. A local-level, north-pointing INS with two ESG's of the same quality (0.0001 deg/hr random drift rate) operating in the pure inertial mode under identical initial conditions and over the same flight path, diverged to a CEP of 2 n. mi. after 41.67 min of flight. The north and east velocity errors exceed 20 fps and 160 fps respectively at this time.

The performance obtained from the direct use of two ESG's in a conventional INS mechanization operating over the 12 hr on-station flight profile is not comparable to the performance obtained using the proposed scheme. Consequently, the cost advantage of the two approaches were not investigated.

Conclusions

The study described in this paper indicates the feasibility of achieving high navigational accuracy for long-duration flights from a self-contained system comprising an ESG inertial reference mounted on the stabilized platform of a local-level INS, and integrated with a Kalman-Bucy filter.

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